

$$T: \mathbb{R}^2 \rightarrow \mathbb{R}^2 \quad T(x, y) = (-x, y)$$

$$N(T) = \{ (x, y) : \begin{aligned} T(x, y) &= (0, 0) \\ (-x, y) &= (0, 0) \end{aligned} \}$$

$$= \{ (0, 0) \} \quad \Rightarrow x=0, y=0 \quad \dim N(T) = 0$$

$$R(T) = \{ T(x, y) : (x, y) \in \mathbb{R}^2 \}$$

$$= \{ (-x, y) : x, y \in \mathbb{R} \}$$

$$T(1, 0) = (-1, 0) \quad \& \quad T(0, 1) = (0, 1)$$

$$\text{Span } R(T) = \{ (-1, 0), (0, 1) \} = \text{Basis of } R(T)$$

$$\dim R(T) = 2$$

$$T(x, y) = (x, 0) \quad T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$$

$$N(T) = \{ (x, y) : \begin{aligned} T(x, y) &= (0, 0) \\ (x, 0) &= (0, 0) \end{aligned} \}$$

$$= \{ (0, y) \} \quad x=0$$

$$\dim(N(T)) = 1 \quad \text{Basis} = (0, 1)$$

$$R(T) = \{ T(x, y) : (x, y) \in \mathbb{R}^2 \}$$

$$= \{ (x, 0), x \in \mathbb{R} \}$$

$$\dim R(T) = 1 \quad \text{Basis} = (1, 0)$$

Rank Nullity thm: $\dim V = \dim R(T) + \dim N(T)$
 $= \text{Rank of } T + \text{Nullity of } T$

Q: $T(2, 3) = (4, 5) \quad \& \quad T(1, 0) = (0, 0) \quad T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$

find Basis & $\dim$ of $N(T)$ & $R(T)$.

Ans Basis of $\mathbb{R}^2 = \{ (2, 3), (1, 0) \}$

$$(x, y) = \alpha_1 (2, 3) + \alpha_2 (1, 0)$$

$$(x, y) = (2\alpha_1 + \alpha_2, 3\alpha_1)$$

$$\Rightarrow y = 3\alpha_1 \quad , \quad 2\alpha_1 + \alpha_2 = x$$

$$\alpha_1 = \frac{y}{3} \quad \alpha_2 = x - \frac{2y}{3}$$

$$\begin{aligned} T(x, y) &= \alpha_1 T(2, 3) + \alpha_2 T(1, 0) \\ &= \frac{y}{3} (4, 5) + \frac{x - 2y}{3} (0, 0) \end{aligned}$$

$$T(x, y) = \left(\frac{4y}{3}, \frac{5y}{3} \right)$$

$$\begin{aligned} N(T) &= \{ (x, y) : T(x, y) = (0, 0) \\ &\quad " \quad \left(\frac{4y}{3}, \frac{5y}{3} \right) = (0, 0) \\ &\quad " \quad y = 0 \end{aligned}$$

$$= \{ (x, 0) : x \in \mathbb{R} \} \quad \dim N(T) = 1$$

Basis = $(1, 0)$

$$R(T) = \{ T(x, y) : (x, y) \in \mathbb{R}^2 \}$$

$$= \left\{ \left(\frac{4y}{3}, \frac{5y}{3} \right) : y \in \mathbb{R} \right\}$$

$$\dim R(T) = 1 \quad \text{Basis} = \left(\frac{4}{3}, \frac{5}{3} \right)$$

Ex: $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ $T(1, 1, 1) = (2, 2)$, $T(1, 1, 0) = (2, 1)$
 $T(1, 0, 0) = (1, 1)$

given Basis = $\{ (1, 1, 1), (1, 1, 0), (1, 0, 0) \}$

$$T(x, y, z) = \alpha_1 (1, 1, 1) + \alpha_2 (1, 1, 0) + \alpha_3 (1, 0, 0)$$

$$(x, y, z) = \alpha_1 (1, 1, 1) + \alpha_2 (1, 1, 0) + \alpha_3 (1, 0, 0)$$

$$= (\alpha_1 + \alpha_2 + \alpha_3, \alpha_1 + \alpha_2, \alpha_1)$$

$$\alpha_1 = z, \quad \alpha_1 + \alpha_2 = y, \quad \alpha_1 + \alpha_2 + \alpha_3 = x$$

$$\alpha_2 = y - z, \quad \alpha_3 = x - y$$

$$\rightarrow T(x, y, z) = \alpha_1 T(1, 1, 1) + \alpha_2 T(1, 1, 0) + \alpha_3 T(1, 0, 0)$$

$$= z(2, 2) + (y - z)(2, 1) + (x - y)(1, 1)$$

$$T(x, y, z) = (x + y, x + z)$$

$$N(T) = \{ (x, y, z) : T(x, y, z) = (0, 0) \}$$

$$= \{ \quad : (x + y, x + z) = (0, 0) \}$$

$$= \{ \quad : x + y = 0 \text{ \& } x + z = 0 \}$$

$$= \{ (x, y, z) : x = -y \text{ \& } x = -z \}$$

$$= \{ (x, -x, -x) : x \in \mathbb{R} \} = x(1, -1, -1)$$

$$\dim N(T) = 1 \quad \text{Basis} = \{ (1, -1, -1) \}$$

$$R(T) = \{ T(x, y, z) : (x, y, z) \in \mathbb{R}^3 \}$$

$$T(1, 0, 0) = (1, 1), \quad T(0, 1, 0) = (1, 0), \quad T(0, 0, 1) = (0, 1)$$

$$\text{Span } R(T) = \{ (1, 1), (1, 0), (0, 1) \}$$

$$\text{Basis } R(T) = \{ (1, 0), (0, 1) \} \quad \dim R(T) = 2$$

Ex: 1) $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ $T(1, 0) = (1, 1)$ \& $T(0, 1) = (-1, 2)$

Construct T \& hence verify Rank Nullity thm.

$$2) \quad T(x, y, z) = (x - y + 2z, \quad 2x + y, \quad -x - 2y + 2z)$$

2) $T(x, y, z) = (x - y + 2z, 2x + y, -x - 2y + 2z)$
Find $R(T)$ & $N(T)$ & their dim & basis.